

Unfolding the Effects of the T=0 and T=1 Parts of the Two-Body Interaction on Nuclear Collectivity in the f-p Shell

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Calculations of the spectra of various even–even nuclei in the fp shell (^{44}Ti , ^{46}Ti , ^{48}Ti , ^{48}Cr and ^{50}Cr) are performed with two sets of two-body interaction matrix elements. The first set consists of the matrix elements of the FPD6 interaction. The second set has the same T=1 two-body matrix elements as the FPD6 interaction, but all the T=0 two-body matrix elements are set equal to zero (T0FPD6). Surprisingly, the T0FPD6 interaction gives a semi-reasonable spectrum (or else this method would make no sense). A consistent feature for even–even nuclei, e.g. $^{44,46,48}\text{Ti}$ and $^{48,50}\text{Cr}$, is that the reintroduction of T=0 matrix elements makes the spectrum look more rotational than when the T=0 matrix elements are set equal to zero. A common characteristic of the results is that, for high spin states, the excitation energies are too high for the full FPD6 interaction and too low for T0FPD6, as compared with experiment. The odd–even nucleus ^{43}Ti and the odd–odd nucleus ^{46}V are also discussed. For ^{43}Sc the T=0 matrix elements are responsible for staggering of the high spin states. In general, but not always, the inclusion of T=0 two-body matrix elements enhances the B(E2) rates.

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I. INTRODUCTION

The study of neutron–proton pairing, especially in the T=0 channel, is a particularly prominent topic these days. While the number of journal articles are far too numerous to reference, one might begin to make some headway into the varied approaches by starting from the references found in Refs. [1, 2, 3]. In so doing one will find a field of study filled with disagreement and occasionally strife.

For example, Macchiavelli et al. [2] claim that some apparent indicators of T=0 pairing can really be explained in terms of symmetry energies. In their abstract they say “After correcting for the energy we find that the lowest T=1 state in odd–odd N=Z nuclei is as bound as the ground state in the neighboring even–even nucleus, thus providing evidence for isovector np pairing. However the T=0 states in odd–odd N=Z nuclei are several MeV less bound than the even–even ground states...there is no evidence for an isoscalar (deuteron like) pair condensate in N=Z nuclei.” While in this work we do not want to get into the arguments between this group and others on this point, we find their work a useful source of motivation for this present study.

In this work we will examine the yrast spectra of the even–even (fp) shell nuclei ^{44}Ti , ^{46}Ti , ^{48}Ti , ^{48}Cr and ^{50}Cr , as well as the odd–odd nucleus ^{46}V . We will perform full fp shell calculations and compare the spectra to experiment. For comparison purposes we also discuss the odd A nucleus ^{43}Sc (^{43}Ti) and the odd–odd nucleus ^{46}V .

In this work we perform the shell model calculations using the shell model code ANTOINE [4]. In order to best see the effects of the T=1 and T=0 interactions, we perform two sets of calculations. In the first we use

the FPD6 interaction [5]. Then we do the same calculations but we set all the T=0 two-body interaction matrix elements to zero. We shall denote this interaction as T0FPD6. (This modification of an effective interaction is along the same lines of that used by Satula et al. to examine Wigner energies a few years ago [6].) We have used this modification of FPD6 in the past to study a variety of things and in particular the full fp spectrum of ^{44}Ti [10, 11, 12, 13].

It should be noted that in Refs. [10, 11, 12, 13] a wide range of topics is addressed beyond the spectra of even–even nuclei. These topics include a partial dynamical symmetry that arises when one uses the T0FPD6 interaction in a single j shell for ^{43}Sc and ^{44}Ti . Also, while using the T0FPD6 interaction, a subtle relationship between the T= $\frac{1}{2}$ states in ^{43}Sc and the T= $\frac{3}{2}$ states in ^{43}Ca can be observed, likewise between the T=0 states in ^{44}Ti and T=2 states in ^{44}Ca . We also considered even–odd nuclei and addressed the topic of how the T=0 two-body matrix elements affect B(M1) transitions—both spin and orbital components, and Gamow–Teller transitions. In many cases the transition rates were very sensitive to the presence or absence of the T=0 matrix elements. This was especially the case for some orbital B(M1)’s and the Gamow–Teller transitions.

Here things will be kept simple and we focus on the spectra and B(E2)’s of the yrast levels of selected even–even nuclei. We will examine the sensitivity of these observables on the T=0 two-body interaction matrix elements by setting them to zero and comparing the results thus obtained with those when the T=0 matrix elements are reintroduced.

The T0FPD6 interaction is not expected to give good binding energies—clearly the T=0 two-body matrix elements are important here. Nor is it expected to give

the relative energies of states of different isospins in a nucleus. This can be partially compensated by adding a two-body monopole interaction in the T=0 channel $a + bt(i) \cdot t(j)$, which for T=0 would be $a(1/4 - t(1) \cdot t(2))$. Such monopole interactions have been studied in the past [7, 8]. However this interaction will not affect the energy differences of states with the same isospin and it will not affect the B(E2) rates.

It should be emphasized that T=0 two-body matrix elements are very important for binding energies. This is especially made clear by the schematic models of Chasman where it is shown that both T=0 and T=1 matrix elements are important in describing the Wigner energy [9]. In this work, however, we are focusing on spectra.

II. DISCUSSION OF SOME PREVIOUS CALCULATIONS

Our entry to this problem considered here was to note that in a single j shell calculation of ^{44}Ti the results for the even J states were almost the same when the T=0 two-body matrix elements of the FPD6 interaction were set equal to zero as they were in a full calculation. This figure is shown in reference [10]. There is an offset of the odd J states. However we point out that none of the odd J states have been found experimentally. In this report we provide an important source of motivation for experiments that should be done.

It should be pointed out that in a single j shell calculation (but not when more than one shell is included) setting the two-body T=0 matrix elements to a constant will give the same relative spectra of T=0 states in ^{44}Ti as will be obtained by setting these to zero.

In another vein we showed that when T=0 two-body matrix elements are set equal to zero one gets a partial dynamical symmetry. For $I = 0$ states of ^{44}Ti with the following angular momenta $I = 3, 7, 9, 10$, and 12, the states can be classified by the dual quantum numbers (J_p, J_n) . However for states with $I = 0, 2, 4, 5, 6$, and 8 no such symmetry exists. We were able to explain this in part by noting that this symmetry exists only for states with angular momenta which are not present for a system of identical particles, i.e. ^{44}Ca .

But even with a full interaction, i.e. when the T=0 matrix elements are present, the T=0 interaction appears to be weak for the states $I = 3, \dots, 12$ for which the dynamical symmetry exists. For example, the wave function of the $J = 3^+$, T=0 state in an MBZ calculation is

$$\Psi = \sum_{J_P, J_N} D^I(J_P, J_N) [(j^2)^{J_P} (j^2)^{J_N}]^I \quad (1)$$

J_P	J_N	3_1^+	T=0	3_2^+	T=0
2	2	0.0000	0.0000		
2	4	0.6968	-0.1202		
4	2	-0.6968	0.1202		
4	4	0.0000	0.0000		
4	6	0.1202	0.6968		
6	4	-0.1202	-0.6968		
6	6	0.0000	0.0000		

The point is that, even with the T=0 interaction present, (J_p, J_n) are almost good quantum numbers. The 3_1^+ state consists mostly of the (J_p, J_n) of (24) and (42); the (46) and (64) amplitudes are only 0.1202. The 3_2^+ state is mainly (46) and (64). When the T=0 matrix elements are turned off, the wave functions collapse to

$$3_1^+ = \frac{1}{\sqrt{2}}[(2, 4) + (4, 2)] \quad (2)$$

$$3_2^+ = \frac{1}{\sqrt{2}}[(6, 4) + (4, 6)] \quad (3)$$

Of course intrinsically the T=0 interaction is not weak but it appears to act weak in certain cases.

III. RESULTS

A. The even-even isotopes $^{44,46,48}\text{Ti}$ and $^{48,50}\text{Cr}$ even J states

In Figures 1 to 5 we show the $T=T_{min}=\frac{|N-Z|}{2}$ even J states of $^{44,46,48}\text{Ti}$ and $^{48,50}\text{Cr}$. In the first column, we have the full f-p shell calculation using FPD6. In the second column, we have T0FPD6, which signifies that the T=0 two-body matrix elements have been set to zero. In the third column, experimental yrast levels are shown [14]. We show separately a comparison of the odd J states in the Ti and Cr isotopes for FPD6 and T0FPD6 in Figures 6 to 10. These figures show experimental even J levels and some odd J levels, although not much is known about the odd J states in these nuclei. Hopefully this paper will serve as an impetus to search for such states.

We will now make some broad remarks about the results. The first point to be made is that with the full FPD6 interaction one gets a very good overall fit to the experimental spectrum. This should not come as a surprise. They were designed to do so.

What is surprising is that, when we set all T=0 matrix elements to zero (T0FPD6), we get a semi-resonable spectrum. If this were not the case, then what we are doing would make no sense. It would appear that the T=1 two-body matrix elements, acting alone, gave us the “spine” of the spectrum. The addition of the T=0 matrix elements gives a needed overall improvement, but as we will show later, there are still some discrepancies even with the full interaction.

I should be pointed out that if we had reversed the procedure and set all the T=1 matrix elements to zero

and kept the T=0 matrix elements as they are, we would get an unrecognizably bad spectrum.

We next take a closer look at the two calculated spectra. We see that with the full FPD6 in ^{44}Ti , the spectra for $J = 0, 2, 4, 6$, and 8 looks somewhat more rotational than with T0FPD6. This is consistent with the knowledge that the T=0 n-p interaction enhances the nuclear collectivity. In the rotational limit the spectrum would be of the form $J(J+1)$ while in the simple vibrational limit one gets equally spaced levels. Experiment resides between these two limits.

Comparing FPD6 to T0FPD6, we find a closer agreement for the even J spectra for ^{46}Ti than for ^{44}Ti . Indeed the closeness in ^{46}Ti is remarkable. It could be that ^{44}Ti is relatively more rotational than ^{46}Ti and hence the T=0 interaction plays a more important role. This could also be a measure of the relative numbers of T=0 pairs in a T=0 nucleus as opposed to a T=1 nucleus.

With one notable exception, the spectrum of ^{48}Ti is as good as that of ^{46}Ti when the T=0 matrix elements are set to zero. The exception concerns the ($J = 6, J = 4$) splitting which is too small when we remove the T=0 matrix elements. This could be connected with the near degeneracy of the two lowest 6^+ levels in ^{48}Ti , a problem that we previously addressed in [15]. In FPD6 these levels are separated by 0.08 MeV and in T0FPD6 by 0.23 MeV. In the single j shell model, the two $J = 6^+$ states have opposite signatures—this might explain in part why there is not a lot of level repulsion between both $J = 6^+$ states.

For the even J states of the N=Z nucleus ^{48}Cr , the low spin spectrum ($J = 0, 2, 4$, and 6) is more in the direction of a rotational spectrum with FPD6 than it is with T0FPD6. At higher spins the FPD6 states are at a higher energy than those of T0FPD6. For example, there is a substantial difference—almost 2 MeV for the $J = 14^+$ state. However it should be noted that the **experimental** value of the energy of the $J = 14^+$ state is in between that of the T0FPD6 and FPD6 calculations and is actually closer to T0FPD6. Similiar results hold for $J = 8, 10, 12$, and 16.

For ^{50}Cr there is a similiar story but the differences are not so pronounced. It is difficult to say for which of the two interactions, FPD6 or T0FPD6, the agreement with the experimental spectrum is better.

As an overview, if we look at the results for all the even-even nuclei, we find that the full FPD6 interaction somewhat goes too far in the description of rotational motion, but T0FPD6 does not go far enough. This is especially evident by looking at the high spin states, which, on the average, are too high with FPD6 but too low with T0FPD6. The experimental energies are between these two limits.

B. Odd J states in even-even Ti and Cr isotopes

We show a comparison between FPD6 and T0FPD6 in Figures 6 to 10 for the odd J^+ excitation energies in

^{44}Ti , ^{46}Ti , ^{48}Ti , ^{48}Cr and ^{50}Cr . We note that the experimental data on odd J is very sparse. In ^{44}Ti there are no odd J , T=0 states identified. There is a known 1^+ state at 7216 keV but this state has isospin 1 and has been associated with the scissors mode state. In ^{46}Ti there are 2 nearly degenerate 1^+ T=1 states at 3731 and 3872 MeV. In the f-p shell model space one can only get one 1^+ state at this energy; one of these must be an intruder state. In general, as stated above, there are not too many odd J , T=| $N - Z$ |/2 states known in the even-even Ti and Cr isotopes. We show in the relevant figures the few that are known.

In ^{44}Ti the ordering of odd J states is the same for T0FPD6 as it is for FPD6. However there is a large overall downward shift. This can be taken care of by a one-body field. When this is done the comparison is fairly good but there are some deviations. The splitting of the $J = 1^+$ and 11^+ states (neither of these sates is present in the $f_{7/2}$ model space) is much larger for T0FPD6 than for FPD6. There is more sensitivity in the odd J spectrum to the T=0 two-body matrix elements than for even J . It would therefore be worthwhile to devise experiments that can find these odd J states.

In ^{46}Ti and ^{48}Ti the deviations between T0FPD6 and FPD6 are not as large as for ^{44}Ti , but there are overall one-body shifts to be taken into account.

It should be noted that there is a simplicity in the spectrum of the odd J states. In all three Ti isotopes, we find that, except for the $J = 1^+$ state, there is a sequential ordering $J = 3^+, 5^+, 7^+, 9^+, 11^+, 13^+$, and 15^+ , which suggests a band structure that should be investigated.

For the odd J states of ^{48}Cr , there is a downward shift in the energies of the states calculated with T0FPD6. For $J=7$, the difference is about 2 MeV. Also, very strangely, the T0FPD6 interaction, for which the T=0 matrix elements are set to zero, gives a better fit to the known (and admittedly incomplete) odd J spin spectrum.

For the odd J states of ^{50}Cr , there is also a downward shift of the energies when T0FPD6 is used as compared to the full FPD6 interaction. With the full interaction, there is better agreement for the $J = 1^+$ state, but not so for the other known states $J = 5, 11, 13, 15$, and 17.

One purpose of this paper is to point out that the data on odd J , T=T_{min} states in even-even nuclei is very sparce and it would be of interest to device means, perhaps with radioactive beams and projectiles which have non-zero spin, of exciting such states and unfolding their systematics.

C. The A=43 spectrum

As an example of an odd A system, consider the case of $^{43}\text{Sc}({}^{43}\text{Ti})$ which has been previously discussed [10]. We here show the figure 11, which shows the difference of the spectra when FPD6 and T0FPD6 are used. For high spins there is a staggering with FPD6 which is not present with T0FPD6. The $J = \frac{9}{2}, \frac{13}{2}$, and $\frac{17}{2}$ states come higher

than the corresponding $J = \frac{11}{2}, \frac{15}{2}, \frac{19}{2}$ states. This staggering is due to the $T=0$ interaction. So, by examining this phenomenon, we could learn something about the effective $T=0$ interaction in a nucleus.

D. The $T=0$ and $T=1$ spectra of ^{46}V

Recent studies and calculations for ^{46}V have been performed by Mollar et al. [16] and Brandolini et al. [17].

In Figure 12 we show a full fp calculation for the odd-odd $N=Z$ nucleus ^{46}V . We show both the $T=0$ and $T=1$ states. The latter are isobaric analog states of corresponding states in ^{46}Ti , so the spectra, where a charge independent interaction is used, as in this case, are identical; thus, the previous discussion here applies.

The full FPD6 fit to experiment for the $T=0$ states is reasonable for the $J = 3^+, 4^+$, and 5^+ but the $J = 7^+$ state is much higher experimentally—it is above the 9^+ state while with FPD6 it is below. We now compare FPD6 with T0FPD6. Clearly the $T=0$ states as a whole are shifted up. This can be resolved by adding the $T=0$ monopole interaction $a[\frac{1}{4} - t(1) \cdot t(2)]$. A downward shift of about 1.5 MeV will make the comparison with FPD6 and experiment much better. It is surprising that, when this shift is made, the $T=0$ states agree well with FPD6 and T0FPD6.

IV. B(E2) RATES

The calculated B(E2) rates in the full fp space for ^{44}Ti , ^{46}Ti , ^{48}Ti , ^{48}Cr , and ^{50}Cr are listed in Tables I to VI. The effective charges used are the standard $1.5e$ for the proton and $0.5e$ for the neutron. The difference in the effective charges from 1 and 0 is intended to take care of the fact that the $\Delta N = 2$ and higher excitations are not present in this model space. The results for FPD6 and T0FPD6 are shown. We also display the ratios of the results for the two interactions.

For ^{46}Ti the reintroduction of the $T=0$ two-body matrix elements causes an increase (relative to T0FPD6) of a factor of two or more for all the transitions considered. So there is evidence here that the $T=0$ matrix elements contribute to the collectivity.

The behavior of ^{48}Ti is very similar to that of ^{46}Ti with two exceptions. The B(E2) for the transition $4 \rightarrow 6$ is clearly peculiar in its behavior, as are the transitions involving the $J = 12$ yrast state. While the reason for this behavior of the $J = 12$ state is not yet clear, the $J = 6$ states of ^{48}Ti have been studied before. The existence of two close lying 6^+ states require us to examine this closer. In Table IV we examine the yrast transitions for these close lying states, finding that it is only for the $4 \rightarrow 6_1$ transition that we get a strong enhancement when removing the $T=0$ matrix elements.

In the ^{48}Cr and ^{50}Cr (Tables V and VI), for the most part, the B(E2)'s are larger when the $T=0$ two-body

matrix elements are reintroduced, but there are some notable exceptions. In ^{48}Cr the $14^+ \rightarrow 16^+$ transition is larger for T0FPD6 than for FPD6, the ratio being 1.029. In ^{50}Cr the ratios for $8^+ \rightarrow 10^+$, $10^+ \rightarrow 12^+$ and $12^+ \rightarrow 14^+$ are, respectively, 2.174, 1.271, 1.116. It was previously noted by Zheng and Zamick [19] that the 10^+ state in ^{50}Cr is not consistent with being a member of the $K = 0$ ground state band, rather it looked like a $K = 10$ state, as noted by Zamick, Zheng and Fayache [20]. This is in agreement with the experimental results of Brandolini et al. [21].

V. CLOSING REMARKS

To partially explain why one gets a semi-reasonable spectrum with T0FPD6, we can look at the spectrum of ^{42}Sc , which consists of one proton and one neutron beyond the closed shell ^{40}Ca . The energy levels have been used to get a single j -shell two-body effective interaction in the $f_{7/2}$ shell, i.e., taking matrix elements from experiment. In this simplified procedure, one makes the association $\langle(j^2)^J V(j^2)^J \rangle = E(J) + \text{constant}$. Note that the constant will not affect the excitation energies or wave functions in this model. Thus, for example, the excitation energy of the $J = 6_1^+$, $T=1$ state relative to the $J = 0^+$, $T=1$ state is 3.122 MeV. So we have $\langle(j^2)^6 V(j^2)^6 \rangle = 3.122 \text{ MeV} + \text{constant}$, etc.

Setting the $J = 0$, $T=1$ energy to zero in ^{42}Sc , the remaining states have the following excitation energies (in MeV):

T=1		T=0	
J	Energy	J	Energy
2	1.613	1	0.611
4	2.815	3	1.490
6	3.122	5	1.510
		7	0.616

Note that the total spread of the $T=1$ states ($(E(6) - E(0))$) is 3.122 MeV. More than three times the spread of the $T=0$ states ($(E(5) - E(1))$) of 0.899 MeV. Thus, we can say that, to a first approximation, the $T=0$ spectrum is almost degenerate, judging by the scale set by the $T=1$ interaction. This would then justify the starting point of setting the $T=0$ matrix elements to a constant. It is easy to show that in this single j -shell model space, if one adds a constant to the $T=0$ matrix elements, it will not affect the wave functions of the states and will not affect the excitation energies of the states which have the same isospin.

It can be seen that the two particle $T=1$ spectrum in ^{42}Sc is quite different from that of a pairing interaction, for which the $J = 2, 4$ and 6 states are degenerate. The fact that the excitation energy of the 6^+ state is about twice that of the 2^+ state indicates that other components of the nucleon–nucleon interaction are present, e.g., a quadrupole–quadrupole interaction. Hence, the $T=1$

spectrum of ^{42}Sc has built into it some aspects necessary for nuclear collectivity.

The above discussion suggests that, in a full f-p calculation, the single j components are sufficiently prevalent so as to get the overall pattern of the spectrum in reasonably good shape. The higher shell admixtures then readjust the spectrum so as to change from what is roughly a vibrational pattern to a rotational one, and here the $T=0$ two-body matrix elements play an important role.

In summary, in studying the problem of the $T=0$ neutron-proton interaction in a nucleus, it may prove more fruitful to begin by removing this channel altogether as was done here by setting all the $T=0$ two-body matrix elements to zero and then reintroducing them, rather than adopting the more common approach of investigating the effects of a pairing interaction separated from the rest of the interaction. This may be especially true in the shell model as the suggestion has been made by Satula and Wyss that it may not be appropriate to separate out a pairing interaction from the rest of the Hamiltonian in a shell model context [18].

An examination of the $T=0$ two-body matrix elements in Figure 13 does not show any obvious simplicity. Their distribution looks just as complex as those with $T=1$ shown in Figure 14. If the $T=0$ diagonal matrix elements were all constant and the off-diagonal matrix elements were zero, we could represent the results by a two-body monopole interaction as $a(1/4 - t(1) \cdot t(2))$. This would be an easy explanation of the insensitivity but certainly it would not be a correct one.

Concerning the future of this subject, it would be of great interest to fill in the missing levels which have been shown in the tables. In particular we have noted that, although there is much data on even spins in the even-even nuclei, there is very little known about the odd J positive parity states. Figures 6 to 10 show some interesting band structure for odd J states. If the levels are found, we can put more constraints on the effective nucleon-nucleon interaction in this region.

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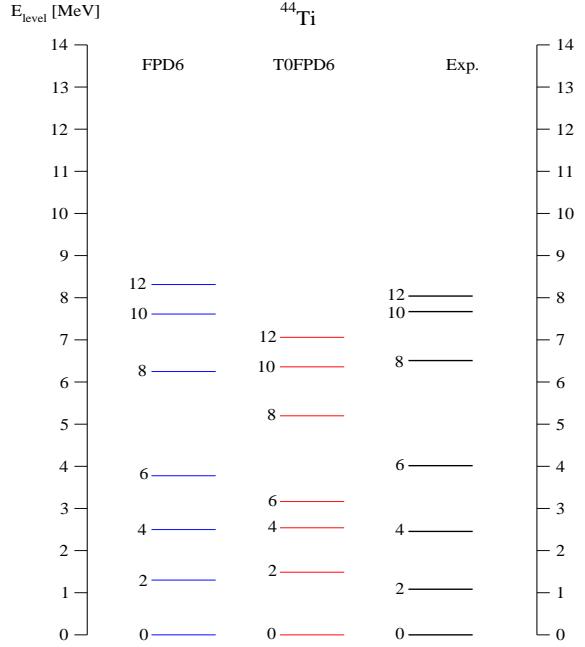


FIG. 1: Full fp space calculations of even J $T=0$ states in ^{44}Ti and comparison with experiment.

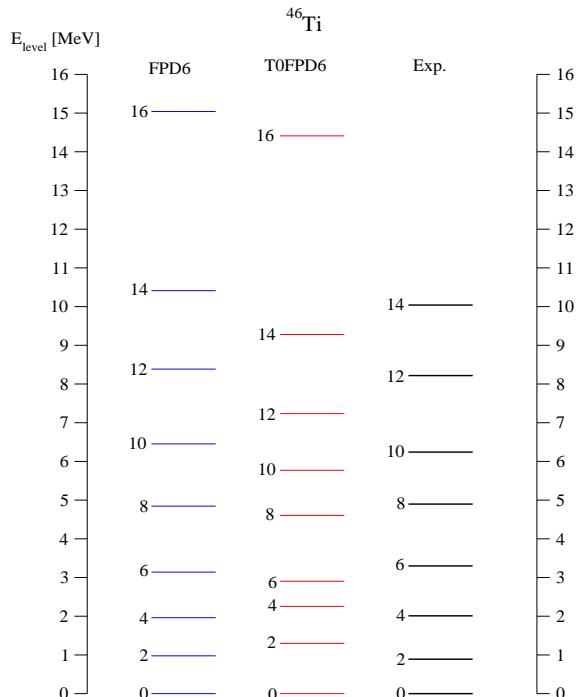


FIG. 2: Full fp space calculations of even J $T=1$ states in ^{46}Ti and comparison with experiment.

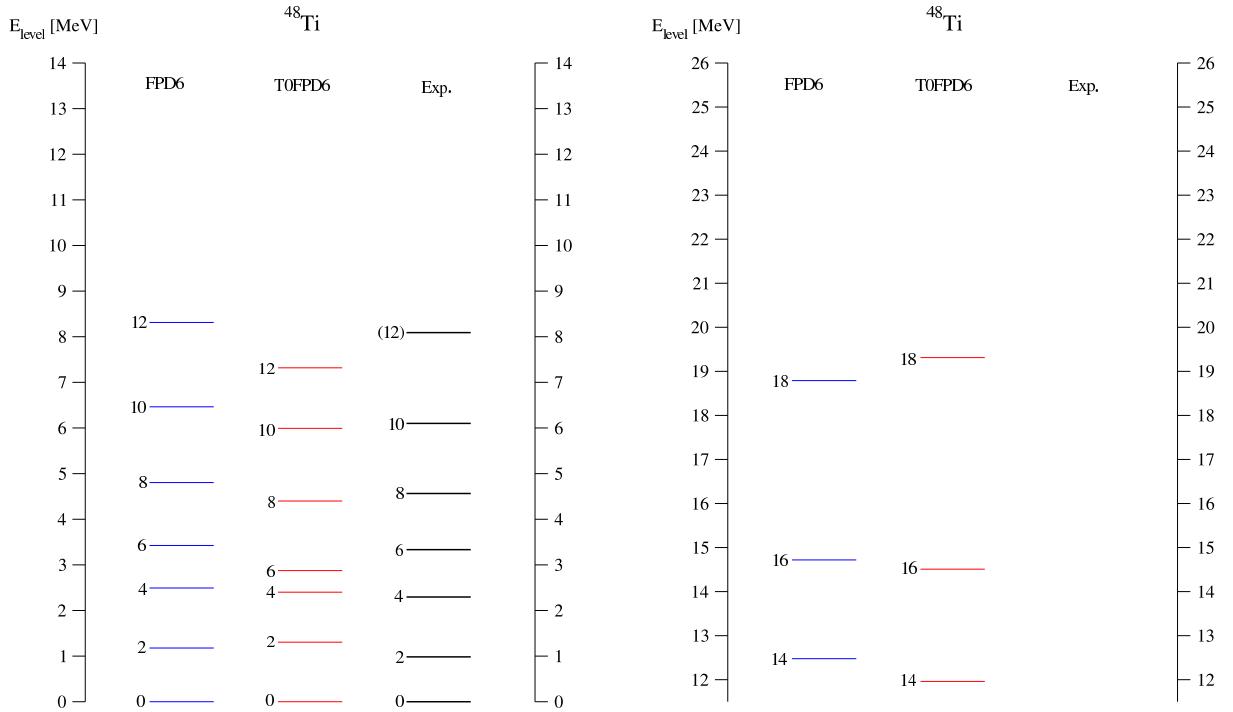


FIG. 3: Full fp calculations of even J $T=2$ states in ^{48}Ti and comparison with experiment.

TABLE I: ^{44}Ti yrast $B(E2)$ values [e^2fm^4] in full FPD6 and T0FPD6

Transition	FPD6	T0FPD6	ratio
$0 \rightarrow 2$	607.24	375.09	0.618
$2 \rightarrow 4$	297.71	146.18	0.491
$4 \rightarrow 6$	202.05	61.164	0.303
$6 \rightarrow 8$	127.20	65.242	0.513
$8 \rightarrow 10$	117.50	78.088	0.665
$10 \rightarrow 12$	65.501	47.968	0.732

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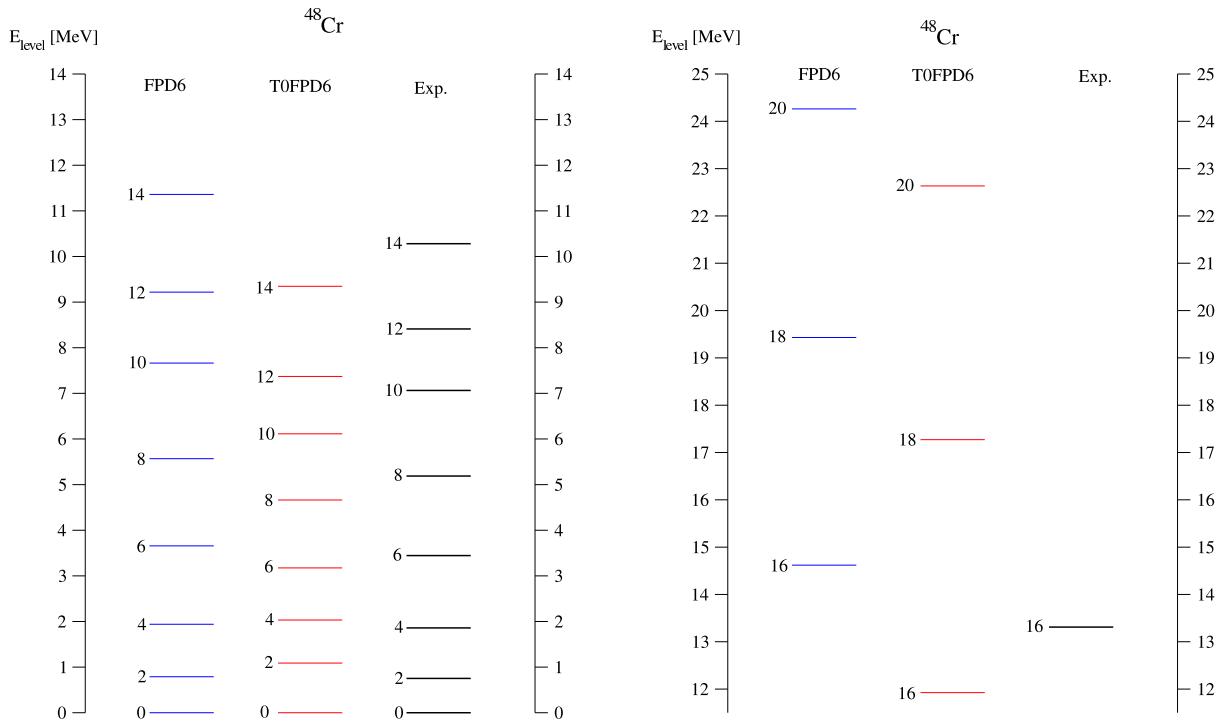


FIG. 4: Full fp calculations of even J $T=0$ states in ^{48}Cr and comparison with experiment.

TABLE II: ^{46}Ti yrast $B(\text{E}2)$ values [e^2fm^4] in full FPD6 and T0FPD6

Transition	FPD6	T0FPD6	ratio
$0 \rightarrow 2$	682.06	432.81	0.635
$2 \rightarrow 4$	349.03	179.18	0.513
$4 \rightarrow 6$	273.85	92.867	0.339
$6 \rightarrow 8$	218.61	82.478	0.377
$8 \rightarrow 10$	157.63	75.154	0.477
$10 \rightarrow 12$	56.441	29.610	0.525
$12 \rightarrow 14$	39.923	18.930	0.474
$14 \rightarrow 16$	1.1333	0.4274	0.377

TABLE III: ^{48}Ti yrast $B(\text{E}2)$ values [e^2fm^4] in full FPD6 and T0FPD6

Transition	FPD6	T0FPD6	ratio
$0 \rightarrow 2$	560.78	401.97	0.717
$2 \rightarrow 4$	306.35	171.89	0.561
$4 \rightarrow 6$	64.147	76.029	1.185
$6 \rightarrow 8$	79.337	26.664	0.336
$8 \rightarrow 10$	75.571	39.341	0.521
$10 \rightarrow 12$	30.055	29.710	0.988
$12 \rightarrow 14$	5.0445	3.4293	0.680
$14 \rightarrow 16$	42.526	11.608	0.273
$16 \rightarrow 18$	0.9308	0.3383	0.363

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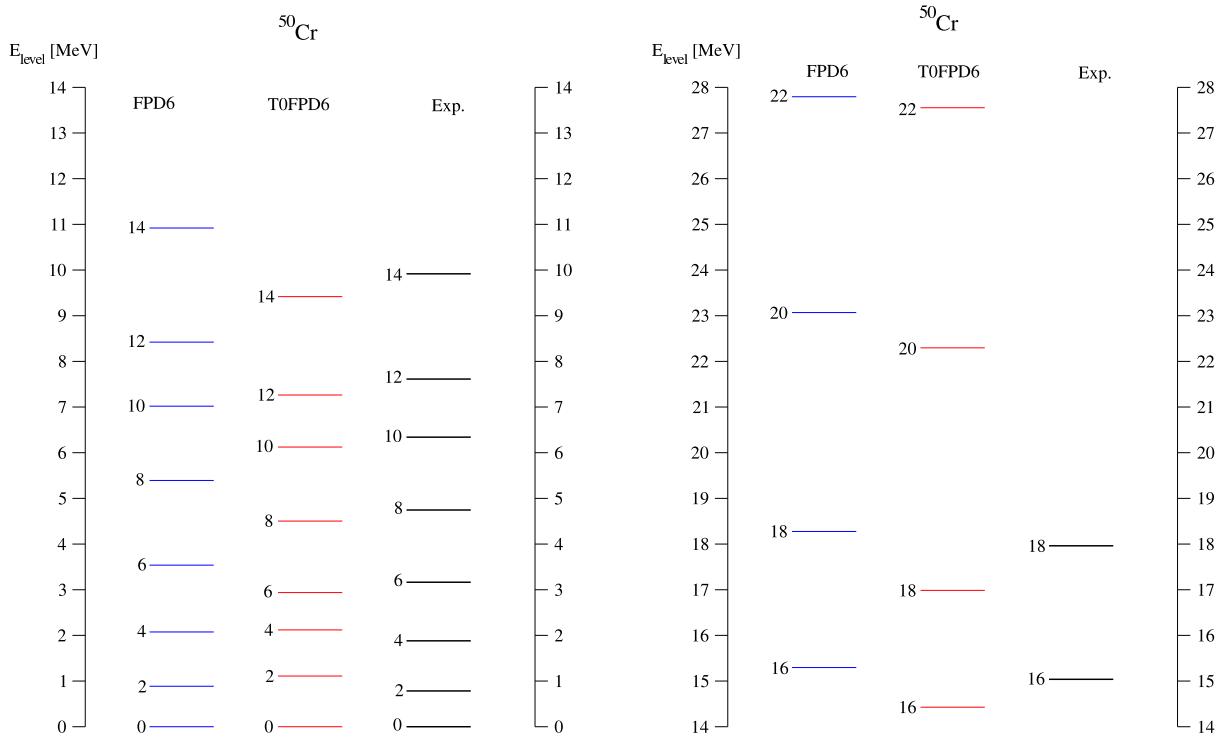


FIG. 5: Full fp calculations of even J $T=1$ states in ^{50}Cr and comparison with experiment.

TABLE IV: ^{48}Ti yrast $B(E2)$ values [$e^2\text{fm}^4$] in full FPD6 and T0FPD6 for 6^+ states

Transition	FPD6	T0FPD6	ratio
<hr/>			
$4 \rightarrow 6_1$	64.147	76.195	1.188
$4 \rightarrow 6_2$	129.29	7.3963	0.057
$6_1 \rightarrow 8$	79.337	26.570	0.335
$6_2 \rightarrow 8$	37.301	14.443	0.387
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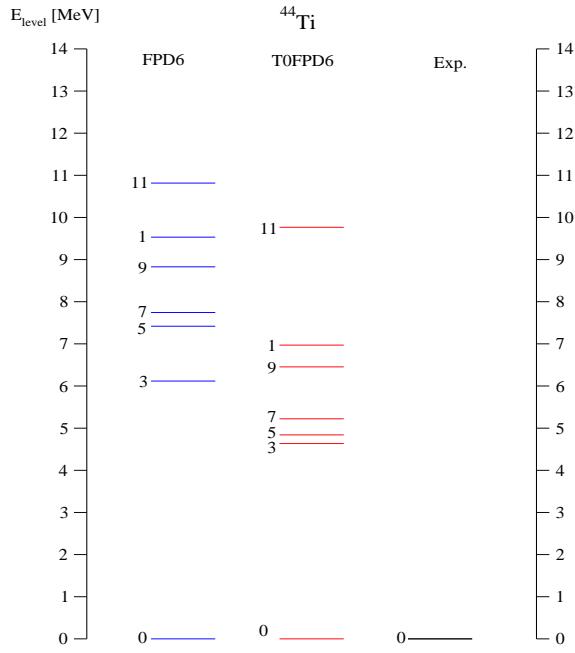


FIG. 6: Full fp space calculations of odd J $T=0$ states in ^{44}Ti .

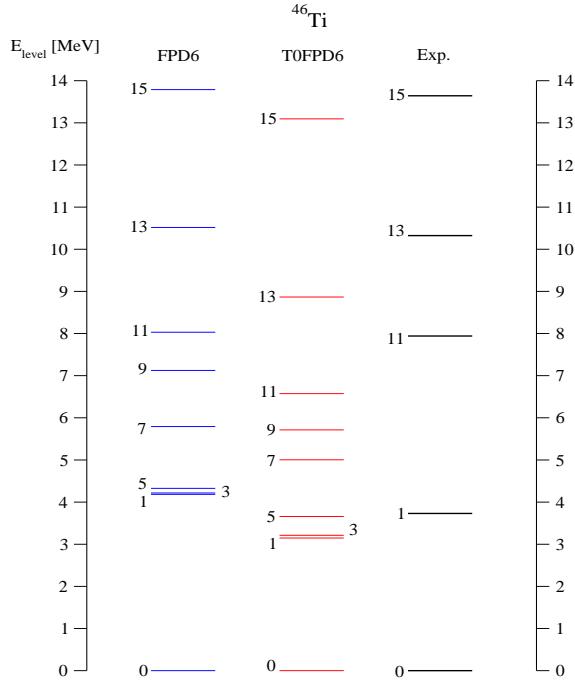
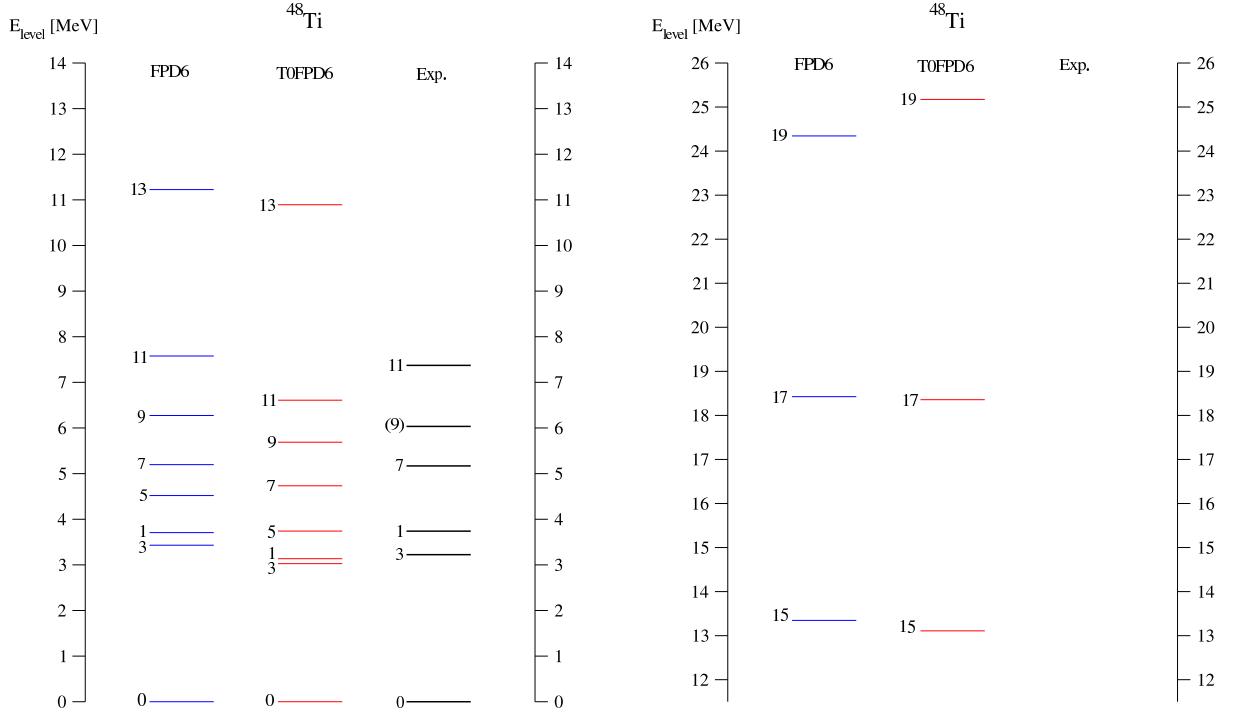
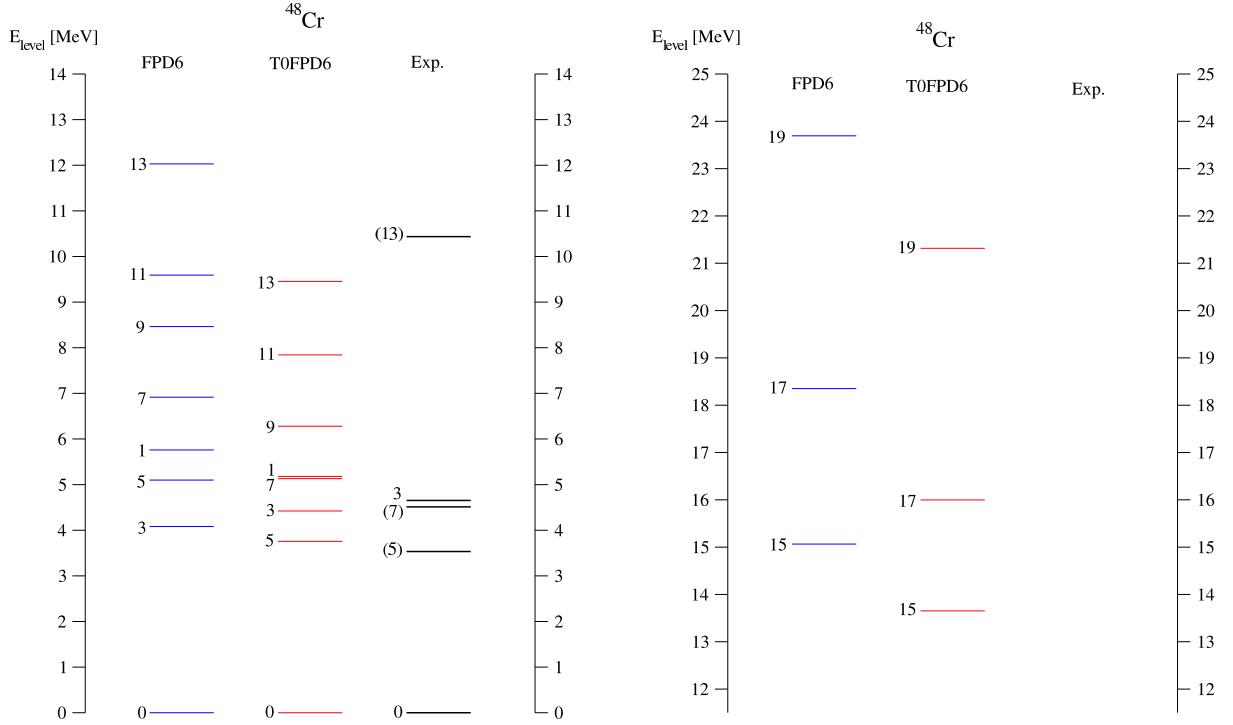


FIG. 7: Full fp space calculations of odd J $T=1$ states in ^{46}Ti and comparison with experiment.

FIG. 8: Full fp space calculations of odd J $T=2$ states in ^{48}Ti and comparison with experiment.TABLE V: ^{48}Cr yrast $B(\text{E}2)$ values [e^2fm^4] in full FPD6 and T0FPD6

Transition	FPD6	T0FPD6	ratio
$0 \rightarrow 2$	1378.4	813.06	0.590
$2 \rightarrow 4$	692.96	376.46	0.543
$4 \rightarrow 6$	577.42	230.16	0.399
$6 \rightarrow 8$	491.87	241.58	0.491
$8 \rightarrow 10$	371.28	194.37	0.523
$10 \rightarrow 12$	157.33	123.28	0.784
$12 \rightarrow 14$	140.42	112.80	0.803
$14 \rightarrow 16$	69.141	71.157	1.029
$16 \rightarrow 18$	1.8306	1.4921	0.815
$18 \rightarrow 20$	7.5903	1.8787	0.247

FIG. 9: Full fp space calculations of odd J $T=0$ states in ^{48}Cr and comparison with experiment.TABLE VI: ^{50}Cr yrast $B(E2)$ values [$e^2\text{fm}^4$] in full FPD6 and T0FPD6

Transition	FPD6	T0FPD6	ratio
$0 \rightarrow 2$	1219.0	736.60	0.604
$2 \rightarrow 4$	636.22	341.01	0.536
$4 \rightarrow 6$	427.64	147.30	0.344
$6 \rightarrow 8$	349.16	156.47	0.448
$8 \rightarrow 10$	36.549	79.449	2.174
$10 \rightarrow 12$	48.488	61.638	1.271
$12 \rightarrow 14$	66.120	73.792	1.116
$14 \rightarrow 16$	4.3417	3.4128	0.786
$16 \rightarrow 18$	85.995	42.408	0.493
$18 \rightarrow 20$	1.8424	0.8246	0.448

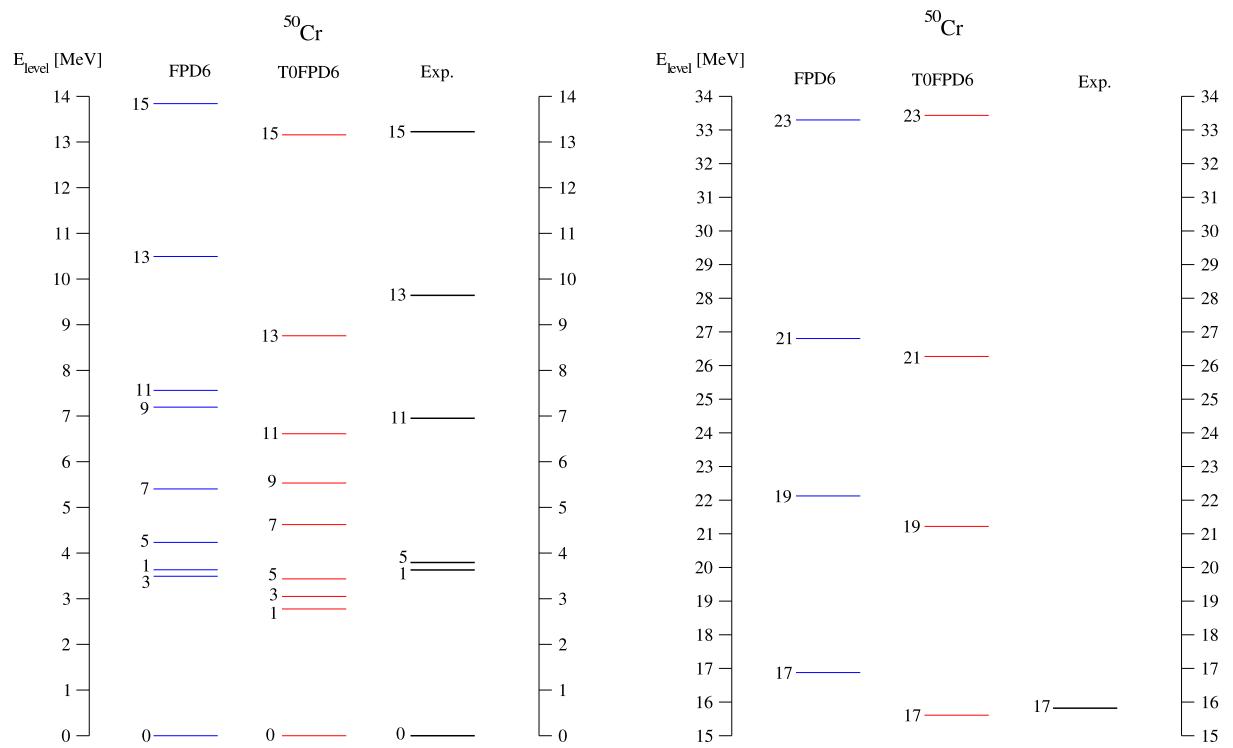


FIG. 10: Full fp space calculations of odd J $T=1$ states in ^{50}Cr and comparison with experiment.

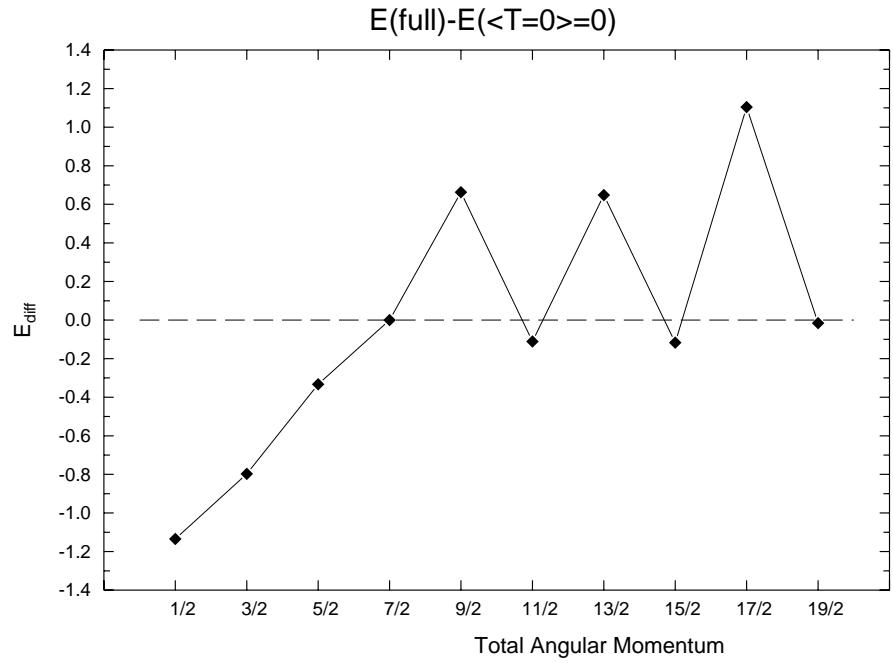


FIG. 11: $E(\text{FPD6}) - E(\text{T0FPD6})$ (MeV) vs Total Angular Momentum (\hbar) in ^{43}Ti .

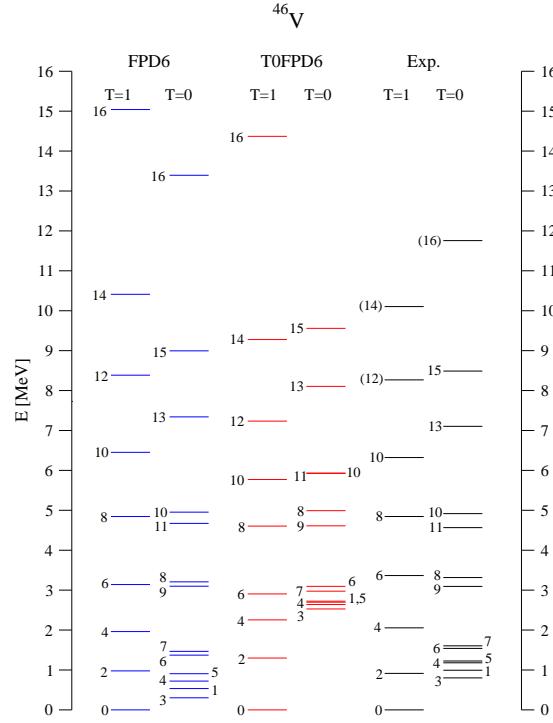


FIG. 12: Full fp calculation and experimental results for T=0 and 1 states in ^{46}V .

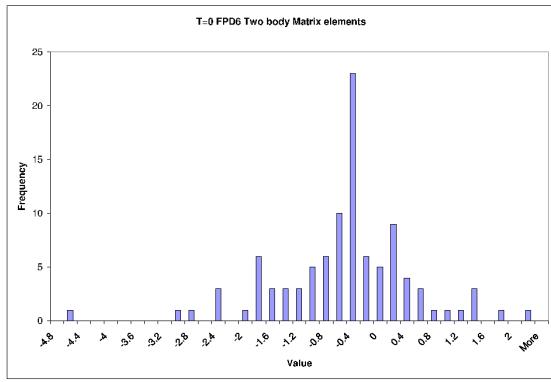


FIG. 13: T=0 two-body matrix element distribution for FPD6

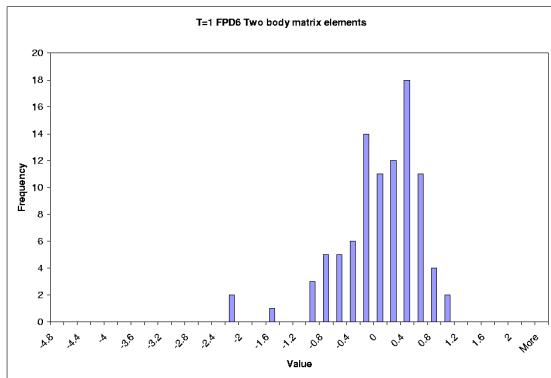


FIG. 14: T=1 two-body matrix element distribution for FPD6